FLOW OF POLYMER MELTS THROUGH CHANNELS OF TRIANGULAR CROSS SECTION

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Analyzed is the problem of a completely steady isothermal flow of non-Newtonian fluids through channels. A close agreement between theoretical and experimental data has been established.

During various technological processes of forming plastic materials and resin mixtures into manufactured parts there occurs a flow of anomalous-viscous fluid through channels of noncircular cross sections [1-4]. The flow conditions do, to a large extent, predetermine the quality of the finished parts.

Statement of the Problem. Basic Equations of Motion and Their Solution. We consider the laminar flow of an anomalous-viscous fluid through a prismatic channel with a cross section in the shape of an equilateral triangle (Fig. 1), assuming that the fluid is incompressible and subject to the rheological law

$$\varphi = f(\tau^2) \tag{1}$$

The problem will be solved with the following assumptions: 1) cross-currents in the channel are negligible, $v_z = v(x, y)$; 2) the flow is isothermal; and 3) no slip occurs at the channel walls. With these stipulations, the equation of motion in terms of stress components becomes

$$\frac{\partial \tau_x}{\partial x} + \frac{\partial \tau_y}{\partial y} = \frac{\partial p}{\partial l}. \tag{2}$$

The continuity equation for the flow is

$$\frac{\partial v}{\partial z} = 0. \tag{3}$$

Multiplying both sides of Eq. (2) by v and integrating with respect to the channel section yields

$$\iint_S \left(\frac{\partial \tau_x}{\partial x} + \frac{\partial \tau_y}{\partial y}\right) v\,dxdy = \frac{\partial p}{\partial l} \iint_S v\,dxdy, \tag{4}$$

where $\iint_S v\,dxdy = q$.

Equation (4) can be written as

$$\iint_S \left[\frac{\partial (\tau_x v)}{\partial x} + \frac{\partial (\tau_y v)}{\partial y}\right] dxdy - \iint_S \left(\tau_x \frac{\partial v}{\partial x} + \tau_y \frac{\partial v}{\partial y}\right) dxdy = \frac{\partial p}{\partial l} q. \tag{5}$$

Replacing the first double integral in Eq. (5) by a contour integral will result in

$$\oint_z v\tau\,dl - \iint_S \left(\tau_x \frac{\partial v}{\partial x} + \tau_y \frac{\partial v}{\partial y}\right) dxdy = \frac{\partial p}{\partial l} q, \tag{6}$$

with z denoting the contour length.

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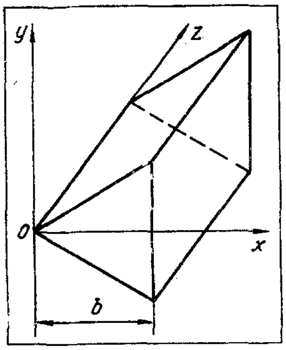

Fig. 1. Schematic diagram of a polymer melt flowing through a channel of triangular cross-section.

Since at the channel wall $v_W = 0$, hence the contour integral is also equal to zero and

$$-\iint_S \left(\tau_x \frac{\partial v}{\partial x} + \tau_y \frac{\partial v}{\partial y} \right) dx dy = \frac{\partial p}{\partial l} q. \tag{7}$$

After substituting for $\partial v/\partial x$ and $\partial v/\partial y$ in the double integral (7), we finally have

$$-\iint_S \varphi \tau^2 dx dy = \frac{\partial p}{\partial l} q, \tag{8}$$

where $\tau^2 = \tau_x^2 + \tau_y^2$.

In order to evaluate the double integral (8), it is necessary to know how τ varies across a channel section and, for this purpose, we let

$$\tau_x = -\frac{1}{2} \cdot \frac{\partial p}{\partial l} \cdot \frac{\partial U}{\partial x}; \quad \tau_y = -\frac{1}{2} \cdot \frac{\partial p}{\partial l} \cdot \frac{\partial U}{\partial y}. \tag{9}$$

Here U is a certain function of x and y, then

$$\tau^2 = \frac{1}{4} \left(\frac{\partial p}{\partial l} \right)^2 \left[\left(\frac{\partial U}{\partial x} \right)^2 + \left(\frac{\partial U}{\partial y} \right)^2 \right]. \tag{10}$$

Inserting expressions (9) into (2), we obtain the Poisson equation

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = -2. \tag{11}$$

From the condition of fluid adhesion, we have at the channel wall

$$U = 0. \tag{12}$$

The solution to the Poisson equation for an equilateral triangle with the boundary condition (12) is [6]:

$$U = -\frac{3}{2} \left(1 - \frac{x}{b} \right) \left(y^2 - \frac{x^2}{3} \right), \tag{13}$$

where b denotes the height of the triangle.

Formulas (8)-(13) yield the volume flow rate as a function of the pressure gradient in the channel, of the channel dimensions, and of $\varphi = f(\tau^2)$.

Design and Operating Principle of the Test Apparatus. The flow of polymer melt was studied on a test stand shown in Fig. 2.

Hot air from the model AK-150 MV compressor was passed through the oil separator 2, the filter 3, the safety valve 4, the check valve 5, and into the tank 7 to a pressure of 150 atm. From there it was fed into the cylinder 8 through a pressure reducer 10 with open valves 13, 15 or through an auxiliary air duct with valve 14. In the latter case, for maintaining the pressure exactly, valve 12 released the excess air into the atmosphere. Pump 6 served for lubricating the compressor cylinders.

A model RS-250-58 device served as the pressure regulator.

In order to widen the range of pressure self-regulation to 140 atm abs. and to improve the precision, the design of this device had been somewhat modified: the shaped end of the valve stem was lengthened by 1.5 mm and the turning torque was reduced to one sixth. The pressure in cylinder 8 and in the capillary viscometer 8a was measured with standard manometers 11 connected to post 9. The apparatus was designed for operation with compressed nitrogen and had provisions for installing a nitrogen tank 7a.

The basic component here was the cylinder 8 with a 0.7 liter capacity (Fig. 3). A socket 3 with a copper tube 4 for the gas intake were fastened onto the top of the cylinder shell 1 by means of a retaining nut 2. A channel 5 was installed at the bottom of the cylinder and the connection was sealed with a gasket 6. The cylinder shell was mounted vertically on a flange 7 and two channel beams.

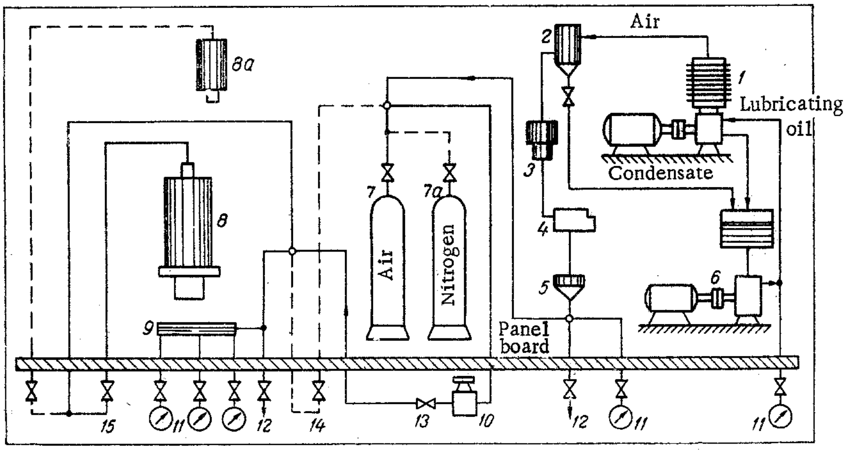

Fig. 2. Basic schematic diagram of the test apparatus.

An annular electric oven 10 with two heater coils was provided for heating the apparatus to the given test temperature. In addition, auxiliary coils 11 and 12 were installed around the upper and the lower part of the cylinder, respectively, for supplemental heating and compensation of heat losses into the atmosphere. The thermal insulation 9 was as thin as possible, the proper thickness having been determined by trial. In order to ensure uniform heating and maintenance of the required temperature, five thermocouples 15 were built in at equal distances covering the entire cylinder height; their hot junctions were all mounted in holes in the wall up to 0.5 mm deep from the inside shell surface. An automatic control and regulation system with a potentiometer 17 actuated a relay 16 and made it possible to maintain the temperature of the inside surface anywhere from room level to 400°C accurately within 1°C. The polymer temperature at the channel exit was measured with thermocouple 15a.

Test Procedure and Test Results. Channels were mounted at the bottom of the cylinder. The potentiometer was set to the required temperature and all heater coils were turned on. After the required temperature had been reached, we checked for uniform heating. For this, by turning switch 18, we measured the temperature at all points and, through autotransformers and ammeters 14, we adjusted the current in all heaters until the temperature became uniform over the entire height of the inside cylinder surface.

The cylinder was then filled 50 cm^3 at a time with polymer pellets and each portion, after having been melted and compressed, was leveled with the floating plunger 8. When the cylinder was full up to the top, the connection to the gas supply system was opened through the pressure reducer 10, by means of which the pressure was stepwise set to various levels while portions of extruded polymer were cut off and then weighed. In this experiment we studied the flow of melted grade P 2015-KU (MRTU 6-05-889-65) polyethylene at 120, 150, and 180°C through channels 47.8 and 98.2 mm long. The channel cross section had the shape of an equilateral triangle (b = 3.8 mm).

In order to compare theoretical and experimental data, the test points were plotted in terms of the mass flow rate Q as a function of the pressure gradient in the channel $\partial p/\partial l$. These points were plotted on paper (Fig. 4) on which the theoretical values of the flow rate according to formulas (8)-(13) had already been marked. The closeness between both sets of results was evaluated in terms of the difference between measured and calculated values as percentages of the measured values. The maximum discrepancy between them, at small pressure gradients $\partial p/\partial l$, did not exceed 40-50% over the entire temperature range.

The double integral in (8) was evaluated by the Senyutovich cubature formula [9], for which one half of the triangular channel profile (Fig. 1) was subdivided into six trapezoids and one right-angle triangle. The values of φ were taken from flow curves for the given polymer material, based on an evaluation of viscosity measurements made with the same cylinder and circular capillary channels (diameter 4 mm, l_1 = 50 mm, and l_2 = 71.5 mm). These data had been evaluated earlier by well-known methods [7, 8].

One phenomenon encountered during extrusion of polymer melts is swelling. Recently processes in free extrusion have been used more often where the part, after it has left the forming channel, is not touched by gages and subjected to tension; the final shape and dimensions depend then on the swelling effect. There are data available in the technical literature [10-13] concerning the swelling of polymer melts extruded from circular capillaries, but no data are available on swelling in noncircular channels. This deficiency has stimulated our study.

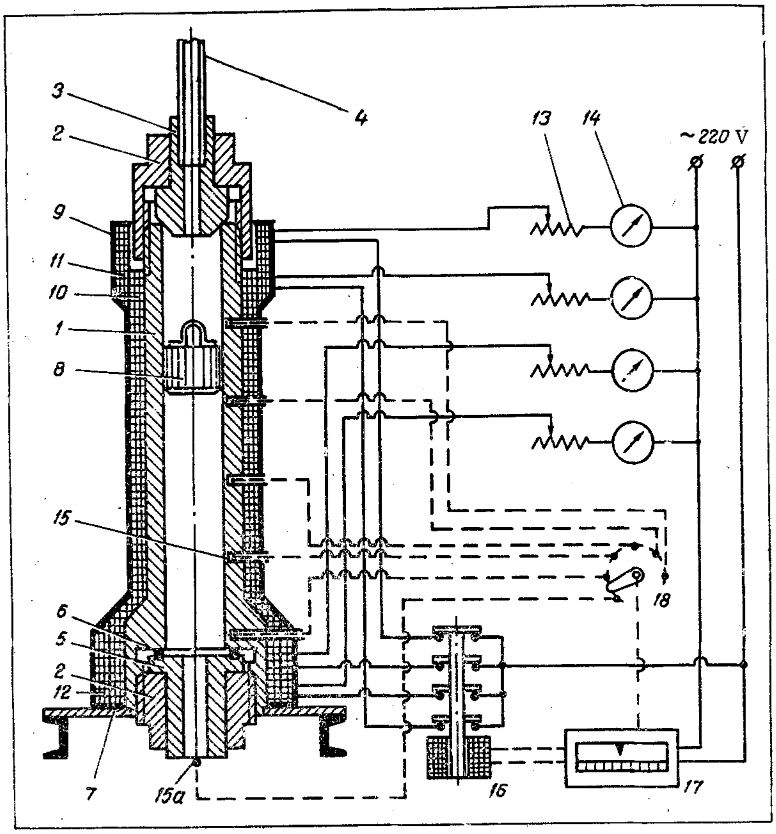

Fig. 3. Cylinder arrangement with a thermostatic system.

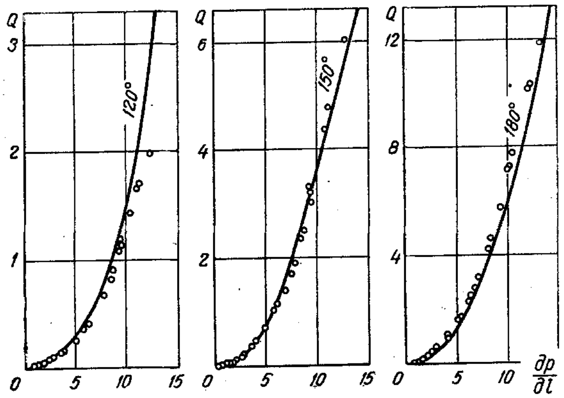

Fig. 4. Mass flow rate of polyethylene Q (g / min) as a function of the pressure gradient in the channel dp/dl (kgf/cm^3): solid lines represent the theoretical solution by the authors; points represent test values.

The swelling of extruded P 2015-KU polyethylene was examined at 120, 150, and 180°C in channels of the following cross sections: a) an equilateral triangle with b = 3.8 mm; b) an isosceles right triangle with b = 2.6 mm; c) a right triangle with one 30° angle and b = 2.6 mm; and d) a scalene triangle with a vertex angle 120°, one acute angle 16°, and b = 1.7 mm. All channel entrance and exit sections were flat and perpendicular to the axis, the edges chamfered.

The test data were evaluated with the aid of a projector. Specimens of extruded material cut under various pressure drops in the channel were sliced transversely into thin wafers (not more than 0.5 mm thick), which were then held squeezed between two flat glass plates and projected with a ×10 magnification on a sheet of millimeter paper. The areas and the circumferences of contours were measured accurately within 1 mm^2 and 1 mm, respectively, whereupon the swelling factor was calculated according to the formula

$$K = d_{e,ex}/d_{e,ch}.$$

The results were plotted on graphs in terms of K as a function of the pressure drop in the channel, for various temperatures and channel lengths. Thermal shrinkage and expansion of extruded material under its own weight were not taken into account.

This relation is shown in Fig. 5 for a channel with an isosceles right triangular cross section. Analogous curves have been obtained for other shapes of the channel cross section. The results of the experiment indicate that the swelling first increases rapidly and then levels off with increasing pressure drops. Characteristically, for all pressure drops and temperatures in all channels studied here, the swelling

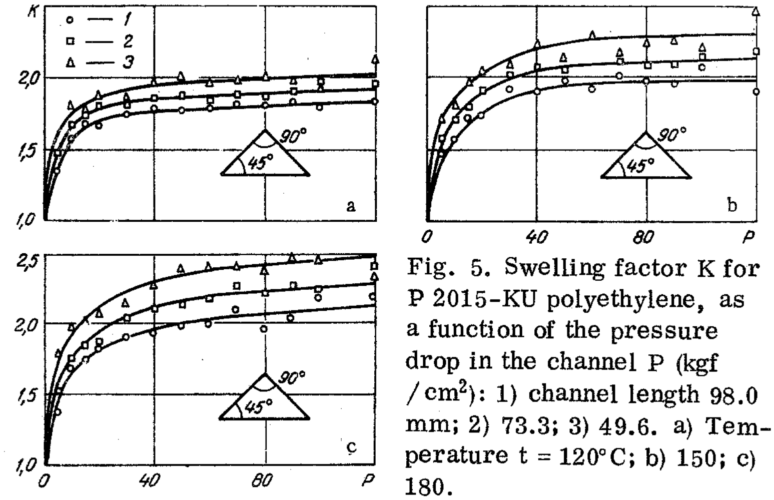

Fig. 5. Swelling factor K for P 2015-KU polyethylene, as a function of the pressure drop in the channel P (kgf $/cm^2$): 1) channel length 98.0 mm; 2) 73.3; 3) 49.6. a) Temperature t = 120°C; b) 150; c) 180.

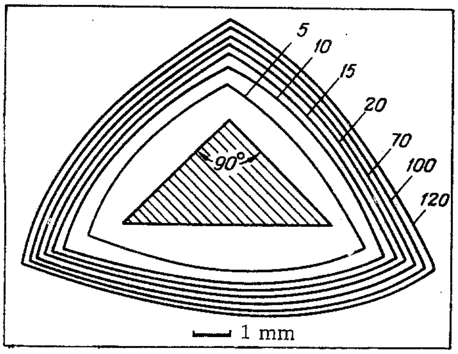

Fig. 6. Swelling diagram for the cross section of P 2015-KU polyethylene extruded at 180°C through a channel 98 mm long.

decreases with increasing channel length at any fixed temperature or with rising temperature at any given channel length and fixed pressure drop.

Changes in the geometry of an extruded shape are shown on the comprehensive swelling diagram in Fig. 6. The channel section at the center is indicated by shading, while the curved lines represent the contours of the material section extruded under 5, 10, 15, 20, 70, 100, and 120 atm abs., respectively. With the aid of this diagram, one can qualitatively estimate the swelling, i.e., the geometrical changes in an extruded section and the effects on it of the extrusion process conditions.

NOTATION

τ is the total shear stress;
τ_x, τ_y are the components of the shear stress;
$\tau_x = (1/\varphi)(\partial v/\partial x)$,
$\tau_y = (1/\varphi)(\partial v/\partial y)$;
φ is the fluidity of anomalous-viscous fluid;
v is the flow velocity;
$\partial p/\partial l$ is the pressure gradient in the channel;
P is the pressure drop in the channel;
q is the volume flow rate of fluid;
Q is the mass flow rate of fluid;
l is the length of channel;
$d_{e,ex}$ is the equivalent diameter of extruded material section;
$d_{e,ch}$ is the equivalent diameter of channel;
K is the swelling factor, dimensionless, $K = d_{e,ex}/d_{e,ch}$;
U is the function, equal to zero on the channel contour;
S is the channel cross-section area.

LITERATURE CITED

1. E. Bernhardt, Processing of Thermoplastic Materials [Russian translation], Moscow (1962).
2. A. N. Levin (editor), Problems in the Extrusion of Thermoplastics [Russian translation], IL (1963).
3. V. M. Kharitonov, G. L. Smirnov, and S. A. Kudryashov, Khim. Volokna, No. 5 (1962).
4. Quick Information, in: Khim. Tekhnologiya Pererabotki Vysokomolekyularnykh Materialov, No. 4 (1966); No. 41, 47 (1968); No. 1 (1969).
5. S. M. Targ, Fundamental Problems in the Theory of Laminar Flow [in Russian], Moscow–Leningrad (1951).
6. L. V. Kantorovich and V. I. Krylov, Approximate Methods of Higher Analysis [in Russian], Moscow–Leningrad (1962).
7. G. V. Vinogradov and N. V. Prozorovskaya, Platicheskie Massy, No. 5 (1964).
8. N. V. Prozorovskaya, Abstract of Dissertation [in Russian], Moscow (1965).
9. V. A. Senyutovich, Trudy Leningradsk. Tekhnolog. Inst. Kholodil'noi Promyshl., 14 (1956).
10. Ya. B. Kaplun and V. S. Kim, Forming Equipment in Extruders [in Russian], Moscow (1969).
11. Ya. B. Kaplun and A. N. Levin, Plasticheskie Massy, No. 2 (1965).
12. A. P. Kiselev and I. F. Kanavets, Plasticheskie Massy, No. 10 (1966).
13. I. S. Klaz and I. F. Kanavets, Plasticheskie Massy, No. 3 (1966).